My Sixty Years along the Path of Probability Theory

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Ladies and Gentlemen, I would like to begin my presentation with the comments I made yesterday at the Kyoto Prize Presentation Ceremony, and then talk about a few memorable episodes from my life, related, in particular, to my 60 years with Probability Theory.

On this beautiful autumn day in my home town of Kyoto, I can think of no higher honor than to have been selected as one of the recipients of this magnificent award.

When I was notified by the Inamori Foundation in June, I was also asked about my current interests and concerns. After a moment's hesitation, I answered: "The future of our planet and of the human race." A few years ago, from around my 80th birthday, I began to write a tale about the future entitled "The Forest People." As I kept telling it over and over to my wife and daughters, after a while they all ended up laughing and saying "Oh no, not that story again about what will happen 20,000 years from now."

The story evolves around the future human race being reborn in the middle of a forest, with a set of values different from ours. Just as *Homo sapiens*, the human species of tens of millennia ago, reappeared on earth after the last ice age as modern man, known as *Homo sapiens sapiens*, so the future human race, many millennia from now, will have survived the even harsher and darker "nuclear winter," and reemerge as *Homo sapiens sapiens sapiens*. We can easily imagine that this three-fold *sapiens*, that is, "wisdom," does not stand for a higher level of intelligence that could avoid making any mistakes, but consists in "human wisdom" in the true sense, not losing, in the face of almost certain defeat by hardship and failure, what gives human beings their humanity—noble goals and love for others.

The Kyoto Prize encompasses a wider range of fields than any other traditional award, and honors those who have contributed to the betterment of humanity through diverse expressions of truth, virtue and beauty—a truly encouraging gift for the future of our planet and of the human race.

Reflecting upon my own work, which was chosen for this prestigious award, all I have done is to write a number of papers describing natural phenomena full of random motions by means of mathematical formulas. I would not be here today, had it not been for those who found in my papers something interesting and some trace of originality, and who extended my work with even more originality to create strict yet beautiful mathematical systems. And in applications beyond pure mathematics, I am indebted to those who built such splendid bridges between the abstract world of mathematics and the real world of nature and human society. I wish to extend my heartfelt gratitude to the Inamori Foundation and to the members of the Kyoto Prize Selection Committee for their generous and scrupulous efforts in awarding this prize to my work over the last fifty years, thereby honoring the enormous contributions of these researchers in mathematics and other fields. Thank you very much.

Human beings now living on earth are called Homo sapiens. It is said that Neanderthal man, also classified as *Homo sapiens*, appeared approximately 250,000 years ago, and prospered in Central Asia and Europe until becoming extinct due to the sudden fall in temperature about 33,000 years ago. Even mammoths, with long hair all over their bodies, became extinct 12,000 years ago before the end of the last ice age. It is remarkable that even though *Homo sapiens Neanderthalensis* disappeared from the earth, *Homo sapiens sapiens* survived and continued to prosper.

After listening to part of my story about the "forest people," a friend of mine asked me if I really believed that a nuclear winter would come, adding "if so, what is the probability of human survival?" Although I study probability theory, I am not a specialist in such a domain, nor am I a specialist in nuclear weaponry. But as a mathematician, my work might be said to be related to the work of such physicists as Albert Einstein and Enrico Fermi. Fermi, the first scientist to build a nuclear reactor, passed away in 1954 when I was a researcher at the Institute for Advanced Study at Princeton, and I still vividly remember how Robert Oppenheimer, the "father of the atomic bomb," was forced to resign as the director of the Institute, because he opposed the production of hydrogen bombs. If a nuclear war should happen on earth, a nuclear winter would surely come, and no human being would survive. Now that the amount of nuclear weaponry is more than sufficient to annihilate not only humans, but also all other species on earth a dozen times over, we might even say that the nuclear winter has in fact already begun. In my story, twenty thousand years from now, human beings will become truly human, having a new set of values, and will begin living in the forest. It may be hard to imagine such a long time ahead, but in light of the 4.5 billion-year-long history of the earth, twenty thousand years is but an instant. After all, the half-life of uranium 238 is 4.5 billion years, and that of plutonium 239 is 24,000 years.

Since I myself coined the term *Homo sapiens sapiens sapiens*, referring to the human species living in the forest 20,000 years from now, I am afraid it is not in any dictionary yet. But many dictionaries do list such terms as *Homo faber, Homo ludens*, and *Homo loquens* as other terms for *Homo sapiens*. *Homo faber* makes tools, *Homo ludens* plays, and *Homo loquens* has the gift of language. Of these definitions of humanity, I have been most attracted to that of *Homo loquens*, which defines the essence of humanity as that of having a language. In everyday conversations with my family and friends, I often pay homage to literary works and musical compositions, but of course my views and comments on such matters are those of a non-specialist, not presentable to such a large audience in such a magnificent hall. What I can say here is that, for mathematicians, mathematics is also a truly beautiful language describing natural laws with strict logic.

If you are interested in my mathematical papers, or in the relationship between my theory and its real world applications, I invite you to the forum to be held tomorrow. Although I had never expected that my theory would be applied to financial operations, leading scientists will give lectures on the “Development of Probability Theory in the 20th Century─From Stochastic Analysis to Mathematical Finance.”

The forest people in my story have a new and simple set of values: they believe that the value of *sapiens*, or true wisdom, must be measured by the number of poets, rather than the number of weapons. It is perhaps clear why they do not evaluate wisdom on the basis of military power. But why poets? Inspired by their superb imaginations, poets create eternal beauty in the form of music or literary works, thereby encouraging and inspiring others. I believe that mathematicians are also true poets just like novelists and musical composers, because inspired by the beauty of previous works, they also try to create new beauty by exploiting their intuition and imagination. I think you will agree with me that when we say “he is a poet,” this statement often implies “he is a strange person,” or even “he is eccentric.” To my regret, when someone says “he is a mathematician,” this also implies “he lacks common sense.” The statement is often used to pardon a mathematician whose mind is solely devoted to mathematical ideas, and who has caused trouble or hurt other people's feelings. As a mathematician, I am afraid I am no exception, and I am grateful that many people have pardoned my mistakes and lack of common sense in my lifetime.

In the category of poets, I would also like to include young people who have lived as Peter Pans for many years, wandering around without aspirations and goals, but who, after a long moratorium, have finally decided to do something with their lives. When they have made their decision, I often find the same sort of dignity in their countenance as I find in that of a poet. In my story of 20,000 years from now, I let these young people and seemingly strange troublemakers enjoy their poetic lives. In twenty thousand years, everyone will suddenly become aware of the value of such poets by the magic wand of the time of the forest.

While stories of prehistoric eras, when history was not separated from mythology, stimulate nostalgic feelings in our minds, many futuristic stories that take place somewhere at the margins of the universe seem to chill our hearts. I believe that ancient stories soften our hearts because many mythological heroes, after long and severe battles, left us beautiful farewell poems or swan songs at their deaths. On the other hand, many futuristic stories are frightening because the heroes of the space age are destined to fall into the crevasses of time and gravity, and seem to be destined to be forgotten forever. As a young boy, I was happily living in the world of Japanese mythology that was often narrated to me by my father, who taught history and Classical Chinese literature in high school and who also compiled two volumes on the local history of Mie.

Near my hometown in Mie Prefecture, there is a hill, where, according to the *Kojiki* (legendary history of Japan compiled in the 8th century), a tragic prince died after a long series of battles lasting over 15 years. Standing on the hill, looking toward his far-away homeland beyond the mountains, he composed four well-known poems and breathed his last breath, when a big swan, his departed soul, soared high into the sky. According to the legend, fatigue and illness made the prince's lower limbs become crooked and three-fold, hence the place name Mie, which literally means three-fold. The hill is at the foot of the Suzuka mountains, not far from a seaside town where I spent my elementary and secondary school days. We would visit that hill on school excursions, and I still recall seeing a calm pond, a thick grove, and a small shrine. It was a very quiet place, and I never saw people there except for us school children. I understand that the Suzuka Circuit (Amusement Park) was constructed near the hill, and I dare not go and see what the hill is like now.

The prince is a hero in Japanese legendary history around the 5th century. A brave warrior, he won many battles in many lands, from Kyushu in the South to Tohoku in the North. With “legs like an antelope, running in the fields and mountains, and a heart like a cloud floating high in the sky,” the prince is remembered as a great hero in all other places in Japan, but he left no heroic deeds worth remembering in Mie. One day, the prince stopped to eat at the shore of Ise Bay, where the rivers Kiso and Nagara merge and flow into the sea. When he left the shore, he forgot his sword under a pine tree. Several years later, his mind and body exhausted, he returned and found his sword leaning against the pine tree. He sang a song of joy, which became a popular Ise folk song. In this song, the prince calls the pine tree “my elder brother,” projecting his nostalgia for his elder brother into the pine tree. Whenever I think of this part of the ancient story, I cannot but feel great sympathy for the prince, imagining how terrible and regretful he felt for having killed his elder brother when he was a young boy.

After the end of World War II, many soldiers returned home and told us how their friends had died in remote battlefields. Their descriptions of their dead friends brought to my mind the ancient prince─modern soldiers also had crooked and three-fold lower limbs, frostbitten or swollen just like a baked rice cake. Although modern soldiers did not die in the forest in Mie, their deaths were very similar to that of the legendary prince. The prince left his homeland to conquer other countries in the name of his father, the emperor. Despite the father’s earnest prayers, the son fell ill in a land far away from his home, and died homesick. Just like the prince, modern soldiers died homesick in an unknown land, some drenched in the rain in a tropical forest, others frozen in an icy conifer forest. Hearing about the last moments of many soldiers, I realized that they had the same father-son relationship that the legendary prince had to his father. To both the modern soldier and the legendary prince, the “father” was a “god,” a supreme commander who loved his “son”─of course, such a supreme father figure is not unique to Japanese thought, but can be found in Christianity as well as in other religions of the world.

Talking of paternal love also reminds me of my personal experience as a young father. During World War II, I was exempted from military conscription because an X-ray examination indicated that I was suffering from tuberculosis. After I returned from the physical examination, my second daughter, who was only four months old, died in a hospital in Tokyo, from whooping cough. At her bedside, I felt so helpless, realizing how futile all my prayers had been. I was 26 years old at the time, and her death left me deeply depressed for a while. It was in February 1942.

Ten years later, a friend of mine from abroad told me a story about forgetting his briefcase at a train station. This is what he told me. When he remembered about the briefcase a few hours later, he went back to the station. To his surprise, he found the briefcase exactly where he had left it. He saw several shabbily-dressed Japanese people watching over it. They said that they were keeping an eye on the briefcase, because they thought this would help the owner more than taking it to the lost-and-found. To my friend, who was very impressed by this kind deed, I proudly said: “That is not surprising at all. In Japan, there is a legend about a sword that a prince left under a pine tree in the 5th century. The sword was still there when he returned several years later. The pine tree is still standing in the town where I was born, and local people still believe in this folklore.”

So, I spent most of my childhood days listening to Japanese myths and legends. In my twenties, I experienced the menace and misery of the war. These early experiences led me to believe that I must oppose any war, under any claim of right. Throughout human history, many wars were fought in the name of god and other divinities, and in the modern age, war slogans instead used the name of “justice” or “democracy.” Whatever the claim of right, wars have become increasingly cruel, and people’s prayers have always proved futile. During the 1950s, when I was in the United States enjoying the intellectual life and material prosperity of American society, I made many international friends, who helped me tremendously with my later research. Given my background, I can only pray that there will be no “nuclear winter.”

In April 1935, upon graduation from high school in Nagoya, I entered the Mathematics Department in the Faculty of Science at the Imperial University of Tokyo (now the University of Tokyo). As I was leaving for Tokyo, a friend of mine handed me the novel *Sanshiro* by Soseki Natsume, saying that the main character of the novel had a life story similar to mine. In those days, it took $5^1/_2$ hours from Nagoya to Tokyo by the super express train, which was more than sufficient to finish the novel. The hero, Sanshiro, went to Tokyo from his hometown in Kumamoto Prefecture on the island of Kyushu in September 1908. I went to Tokyo 26 years later, but I felt the same culture shock as Sanshiro, and also tried to soothe my exhausted mind by the pond on campus, just as he did in the novel. This pond is now called Sanshiro Pond. I believe that the friend who gave me the book also imagined that I would encounter various troubles in an unknown place. However, I must say that I was much more fortunate than Sanshiro because Tokyo was where I met my lifelong mentors and my lifelong partner.

The faculty of the Mathematics Department at the time included Professors Takagi, Nakagawa, Takeuchi, Suetsuna, Tsuji, Kakeya, and Iyanaga. In 1936, one year after I entered the university, we experienced the February 26th Incident (a failed military coup d'etat in Tokyo in which several political figures were killed), followed in 1937 by the Marco Polo Bridge Incident (a clash between Japanese and Chinese troops triggering the Sino-Japanese War of 1937—1945), leading into World War II. On campus, however, we were allowed to devote ourselves to our studies and research in a quiet environment. There was a room for students of the Mathematics Department on the third floor of a building near the Yayoi Gate. Mr. Ishida, an elderly janitor, always had boiling water on the stove, so that he could serve us tea whenever we wanted.

I am often asked how I became interested in probability theory. During my high school days, I was actually interested in mechanics and in fact wanted to study mechanics at the university. I was fascinated by the lectures on free fall and motion in a parabolic curve in the basic mechanics course in high school, and was keenly interested in mathematical descriptions of natural phenomena. I leaned that Kepler's laws of planetary motion were deduced by solving the equation of motion, based on Newton's principles of mechanics and his law of universal gravitation.

At the university, I was stimulated by lively discussions, and was fascinated by the crystal-like structural beauty of pure mathematics. I began to understand that many mathematical concepts have their grounds in the mechanics of natural phenomena, and learned that there was a new type of law called "law of large numbers," which deals with seemingly random phenomena. I thought that knowledge of such laws might help my studies of statistical mechanics, and began reading books on the law of large numbers by Jakob Bernoulli and the central limit theorem of de Moivre. I had learned German and English in high school, and during the first summer vacation at the university I took a three-week intensive French course at a private language institute, and later taught myself Russian. These short periods of study enabled me to read mathematical books in French and Russian. I was able to do this without lengthy language studies because the language of mathematics itself is universal.

This was how I gradually approached probability theory from statistical mechanics. At that time, there were no mathematicians in Japan specializing in probability theory. I felt quite isolated, and furthermore I was doubtful whether probability theory was an authentic mathematical field. It was my mentor, Professor Shôkichi Iyanaga, who encouraged me to specialize in probability theory. He stimulated my initial interest in this field, and I would not have been able to contribute to the development of probability theory had it not been for his support and kind encouragement throughout these years. Although Professor Iyanaga specializes in the theory of numbers, he has a broad perspective and foresight into various other mathematical fields. sixty years have passed since my student days, when I was grappling with various mathematical issues while watching the surface of Sanshiro Pond on campus. Professor Iyanaga, now over 90 years old, still guides me, sometimes even showing his concern for my unsteady gait.

In 1938, upon graduation from the university, I joined the Statistics Bureau of the Cabinet. Secretariat, where I worked for five years until I became associate professor at Nagoya Imperial University. Since I had gotten married as soon as I graduated from the university and my first daughter was born the following year, I could not remain at the university to devote myself to academic research. I was, however, determined to continue studying mathematics, and my five years at the Statistics Bureau were very crucial in developing my career.

Ever since I was a student, I have been attracted to the fact that statistical laws reside in seemingly random phenomena. Knowing that probability theory was a means of describing such phenomena, I began reading papers and books on probability theory during my third year at the university. Gradually, I understood that statistical laws partake of mathematical essence, but I was not satisfied with the contemporary papers on probability theory, since they did not clearly define the random variable, the basic element of probability theory.

Although it is now taken for granted that mathematical systems should be built on strict definitions of each element, this was not common throughout the field until fairly recently. Differential and integral calculus, for instance, had developed into an authentic mathematical system at the end of the 19th century, when a clear definition of the real number concept was formulated. Fortunately, I was able to study differential and integral calculus as an authentic mathematical system, and hear the wonderful lectures by Professors Kondo and Takagi. However, papers and works on probability theory at the time used intuitive descriptions just like 19th century calculus.

Soon after joining the Statistics Bureau of the Cabinet Secretariat, when I was still grappling with the question of how to define the random variable in probability theory, I found a book written by the Russian mathematician Kolmogorov. Realizing that this was exactly what I had been looking for, I read through the book in one sitting. In *Grundbegriffe der Wahrsheinlichkeitsrechnung* (Basic Concepts of Probability Theory), written in German in 1933, Kolmogorov attempted to define random variables as functions in a probability space, and to systematize the theory of probability in terms of the theory of measures. I felt as if this book cleared the mist that was blocking my vision, leading me to finally believe that probability theory can be established as a field of modern mathematics.

In my mind, the foundation of probability theory was thus developed. However, I was faced with another challenge. Contemporary research of probability theory was conducted mainly by studying the behavior of various sequences of independent random variables, with the aim of laying mathematical foundations for statistical laws. This corresponds to the theory of series in the foundation of calculus. Although such studies of statistical laws were more difficult and had more variety than those in the theory of series, at that time, I still thought they were less attractive than other fields of mathematics, and did not feel like devoting myself to the study of statistical laws.

What finally and decisively drew me to the world of probability theory once again was reading *Théorie dé l'addition dés variables aléatoires* (1937) (Theory of the Sum of Independent Random Variables) by the French mathematician, Paul Lévy. This was the first giant step in the study of stochastic processes, placing the concept of a stochastic process as something corresponding to a function in calculus. Having found the essence of the new probability theory in Lévy's paper, I decided to walk the path that Lévy had paved and illuminated with a streak of light. This was in November 1938, exactly 60 years ago.

In the sample functions of stochastic processes in Lévy's theory, I discovered a beautiful structure worthy of being called a mathematical theory. In addition, I was able to learn about Wiener processes, Poisson processes, and processes with independent increments. I was interested in the decomposition theorem for processes with independent increments, which is the core of Lévy's book. His arguments, however, were extremely difficult to follow, since Lévy, a pioneer in this new field of mathematics, explained probability theory based on his intuition. Fortunately, I knew that the American mathematician J. L. Doob introduced the concept of regularization in his paper of 1937, and I thought that I might be able to clarify the fuzzy points in Lévy's arguments by using Doob's concept. In other words, I attempted to describe Lévy's ideas from Doob's viewpoint, and by employing the Poisson random measure, I finally restated Lévy's description of the decomposition theorem as a definite and logical description. This theorem, now called the Lévy-Itô theorem in probability theory, was developed in my first paper, accepted for publication on 1 August 1941, appearing in the 1942 issue of the *Japanese Journal of Mathematics*. This was also my doctoral thesis, and I was granted a doctorate from the Imperial University of Tokyo four years later, on 3 October 1945. I still remember that it was the first clear autumn day after the end of the war, and the sun was shining on the burned ruins that extended as far as I could see.

I was very fortunate to have enjoyed the company of numerous extraordinary and celebrated mentors and many wonderful friends, but probability theory was not popular at that time, and I had to start my career during the war, the epoch that every Japanese calls "that miserable period."

In 1942, I published my second paper, on the differential equation determining a Markov Process. Rather than giving you a detailed explanation of its contents, I would like to tell you an episode related to this paper, which, as far as I know, was only read and understood by one other person.

I published this second paper in the *Journal of Pan-Japan Mathematical Colloquia*, a mimeographed journal issued by Osaka University to encourage young Japanese mathematicians to circulate and exchange their ideas. I hope that some of you in the audience can recall mimeographed copies with nostalgia. The journal was issued during the war, but I did not hear any reaction from other mathematicians about my paper until long after the end of the war, when Gisiro Maruyama told me in person that he had been drafted into the army and had read my paper in the barracks in 1942. Perhaps he and I were the only researchers then interested in the problem of sample paths. Maruyama later developed his ideas based on my theory, and in 1956 published a paper on the topic in the *Rend. Circ. Mat. Palermo*, which I read with great enthusiasm in Princeton.

As I said earlier, when my first and second papers were published in 1942, I was working at the Statistics Bureau of the Cabinet Secretariat. You may wonder how I found time to study and do research, while working as a public government employee. To tell you the truth, I did not do any public employee work to speak of Director Takahiko Kawashima, then Head of the Bureau, told me from the beginning that, since my research is related to the tasks of the Statistics Bureau in a broad sense, I could use all my time to study whatever interested me. Thanks to the generous consideration given to me by the Director, I could concentrate my efforts on the study of probability theory although I did not belong to any university or research institution at the time. I would like to add that Director Kawashima, who passed away some years ago, is the grandfather of Princess Akishino.

I further developed the theory that I had published in my 1942 paper "Differential Equations Determining a Markov Process." Several years later, when I completed my manuscript on stochastic differential equations, I found that no mathematical journal in Japan was willing to publish such a long manuscript because of a general paper shortage under the dismal post-war economic conditions. So I sent my paper to Professor Doob, asking if there was a possibility of publishing it in the United States. Thanks to Professor Doob's kind assistance, the paper was published as a volume of the *Memoirs of the American Mathematical Society*.

The stochastic differential equation I developed in this paper is now called "Itô calculus" and is currently widely used to describe phenomena of random fluctuation over time in both pure mathematics and non-mathematical fields including physics, engineering, biology, and economics. The Itô calculus differs from conventional calculus in that it uses new random parameters accumulated over time as the Brownian motion increments to describe developing phenomena. It now has applications in biological descriptions of phenomena associated with random fluctuations in quantity, and engineering analyses of phenomena that have significant "noise" parameters.

I was a member of the faculty at Nagoya University from 1943 until 1952, when I joined the mathematics faculty at Kyoto University. Although my Nagoya days coincided with the war and the turbulent postwar period, I was very lucky to have had the opportunity to work closely with Professor Kôsaku Yosida, who passed away a few years ago. In my papers, I frequently use the terms "Yosida's theory of semi-groups" and "theory of Hille-Yosida." Both inside and outside the world of mathematics, Professor Yosida had a profound influence on my life, which goes far beyond what I could even hope to describe within the limited time of today's presentation.

In addition to Professor Yosida, I met many people in the past 60 years of my study of probability theory. I learned countless valuable ideas from them, and developed my theory based on their ideas. I had originally intended to talk about these developments, but as I was preparing this speech, I realized that such topics may not interest the general audience. I have written about these developments in the Foreword to my selected papers volume published by Springer (*Kiyosi Itô Selected Papers*, 1987), and I believe they are only of interest to those who are planning to read my papers.

Of the 27 years between 1952 (when I first became professor of mathematics at Kyoto University) and 1979 (when I retired from the Directorship of the Research Institute for Mathematical Sciences at Kyoto University), I spent more than ten years at institutions in the U.S.—Princeton, Stanford, Cornell—as well as three years at Aarthus University in Denmark.

Even after my retirement from Kyoto University, I was able to continue to walk along the path of probability theory at Gakushuin University, ETH in Switzerland, and at the University of Minnesota. Wherever I went, I was able to enjoy walking through the beautiful world of mathematics, thanks to my wonderful teachers and insightful colleagues, as well as my thought-provoking students. Since I cannot possibly do justice to all of their contributions here, I would like to tell you about the *Encyclopedic Dictionary of Mathematics* published by Iwanami.

My mentor, Professor Shôkichi Iyanaga was the principal editor for the first 1954 edition, the enlarged 1960 edition, and the second 1968 edition. I took on the responsibilities of the principal editorship for the third 1985 edition. As I mentioned in the preface to the third edition, the English version of the second edition, published by MIT Press in the U.S., was highly praised throughout the world. Seventeen years after the publication of the second edition, however, even this internationally acclaimed edition needed updating, due to the remarkable progress in mathematics, and the recent development of relations between various mathematical fields, which promoted the organic integration of mathematical studies. Moreover, mathematical theories are now in wide use in various other sciences, and many researchers in non-mathematical fields have begun to regard mathematics as the foundation of all sciences. In response to such a new environment, we decided to publish the third edition by revising the second edition. Upon the completion of the third edition, an English version was immediately prepared and published by MIT Press.

Sixty years ago, when I began my research on probability theory, mathematical papers were mostly written in German, French, Russian, and English. My 1942 paper, “Differential equation determining a Markov Process” was written in Japanese because it was during the war, when the Japanese government prohibited the use of English. In later years, however, I wrote over 60 papers in English. Since I later rewrote my 1942 paper in English, all my papers are now available in English. Today, English is used in almost all mathematical papers and presentations throughout the world, and it appears that English is the lingua franca in other scientific fields as well. As the editor, I therefore note with pride that a highly reliable mathematical dictionary was first compiled in Japanese and subsequently translated into English, and that many scientists throughout the world were awaiting the English version─a testimony to the high level of mathematics in Japan. I would like to thank once again all those who contributed to the success of this encyclopedic dictionary of mathematics.

Although I believe that the Japanese language is highly efficient as a written language, as a spoken language, it has too many homonyms and needs more explanation than other languages to clarify logical connections. When mathematicians discuss something, they always prefer using a blackboard or a sheet of paper to write down the mathematical formulas. Since mathematics is a universal language, mathematicians can communicate with each other quite easily. Even so, first-year-students studying at a university are likely to find it difficult to understand what their professors are saying. Moreover, some of the celebrated professors I knew at the Institute for Advanced Study in Princeton, for instance, gave lectures in the same manner as they wrote papers.

Dr. André Weil, a laureate of the 1994 Kyoto Prize, was one of these professors. After filling the blackboard with mathematical formulas, Professor Weil often erased everything, saying that he made a mistake. Then he left the classroom to think about the problem again in the hallway, and then he came back to continue his lecture. His lectures were very unique and enlightening. Yet, it was extremely difficult to understand everything he said in the classroom. After returning home, I often reviewed my notes to recall his lectures. Going over the lecture notes, I often felt as if I heard the beautiful music of mathematics. Unfortunately, the music sometimes abruptly stopped or faded away. But having access to our mathematical dictionary can sometimes help replay the music. For my own work, the mathematical intuitions, gained watching the water in a pond or a cloud in the sky, had to be developed into strict mathematical logic, and on such occasions many achievements of my predecessors provided me with a valuable index with which to maintain a balance between intuition and logic. By placing their work into the dictionary, I hope that the dictionary will help young researchers by providing them with an index.

In recent years, researchers in various non-mathematical fields have begun developing my mathematical theory in their own original ways, applying it to various other fields. From my mathematical musical scores, they must have found some notes that I did not intend to include, added new phrases, and arranged my original pieces for other instruments that I did not even think of. Their instruments, however, are not magical tools for playing the music of mathematics that can be appreciated by everyone. I truly appreciate their efforts and their new “musical-scores” that bridge the abstract mathematical world with the real world. So long as my theory is used in scientific fields, where each field is organically linked with mathematics, I am extremely happy and grateful to these researchers. When I heard about the use of the “Itô Theory” in the financial world, I was also very surprised but this time somewhat concerned. In the autumn of 1997, I received many letters from my American friends, stating that using the “Itô Theory” in financial operations is now common practice. Then, at the end of 1997 a TV crew came to see me from Tokyo, and also informed me of similar news. One of the letters from thy friends reads as follows:

> In America, math students have changed a lot. In the past, excellent students in math became mathematicians. Now, they become brave “soldiers” in the economic war. In this severe economic war, American soldiers are armed with radar in the form of the “Itô Theory” to shower the enemy with shells. On the other hand, Japanese soldiers still depend on their intuition and spiritual strength to counterattack Americans. They also eat blueberries to enhance their night vision. Although the Japanese army was predominant until a few years ago, they are now losing and fleeing from the battlefields. Having enjoyed cordial friendship with you and many other Japanese mathematicians for half a century,

> we, American mathematicians, are very concerned about this war. In my view, Japanese soldiers will find the radar of the “Itô Theory” before they are entirely defeated and will begin a counterattack, which will further escalate the war.

Then my friend talks about the four Asian dragons, which once spread their wings all over the world, and explains how the dragons are currently withdrawing to their home countries I will omit this part, and tell you the last part of his letter:

> Fortunately, or unfortunately, the “Itô Theory” is just radar, not all atomic bomb. Since radar does not have the power of an atomic bomb, it cannot end the war as promptly as the atomic bomb did. At this moment, no one knows how the war will end, and what the postwar period will be like.

Reading this letter, I was stunned because I had never expected my theory to be used in an “economic war.” In my lifetime, I have never bought any stock, much less their derivatives. I deposit money only in an ordinary bank account, and I have rarely even had a fixed deposit account. According to my wife, when there was a lot of difference in the interest rates between ordinary and fixed deposits, we had no money to deposit. Now that we have a little money, the interests rates are minuscule. So my wife says that it was smart that we paid so little attention to the interest rates. I began writing an answer to my American friend, pointing out the mistake in his statement that Japanese soldiers ate blueberries to enhance their night vision. I wrote, “what they ate during World War II was lamprey eel, not blueberries.” Then I realized that I was writing about such nonsense compared to my friend’s lively report, comparable to Tolkien’s tales of the Hobbit. Instead of commenting on blueberries, I decided to write about recent Japanese students.

> In my student days, excellent Japanese math students became mathematicians, just as in the U.S. All my classmates became mathematicians, and their names are listed in the *Encyclopedic Dictionary of Mathematics*. Today, however, few high school students who are good at math specialize in mathematics. Gold medallists in the so-called Mathematical Olympics, for instance, go on to study

> medicine rather than mathematics. I often say to my friends: “Even if you can solve a given problem quickly, this does not mean you will become a good researcher. To become a good researcher, you must be able to find your own problem, and solve it in your original way, which sometimes takes many years. To be a good researcher, you must enjoy your work.” My view is that young students of today are too smart and quick, and therefore have never thought about a problem for more than 30 minutes.

Since I had never seen a “field of financial operations, where the Itô Theory is used as common practice,” I searched the TV programs by going through the channels. I came upon some video footage of a stock exchange floor, and saw a lot of computer screens covering the walls, just like a scene from a space station. I saw people moving about and shouting. Even though I did not understand what they were doing, I could imagine that they must be making many important decisions at every instant, referring to a huge amount of information. I heard that a delay of three minutes, or even three seconds in decision making could cause a loss of billions, or even trillions of dollars. The soldiers certainly did not look like to have sufficient time to use the simulation of the Itô Theory. I began to doubt if the scene I saw on TV really represented “a battlefield where the Itô Theory is used as radar.”

Then recently I received another letter, this time not from one of my old American friends but from one of my young relatives, currently studying financial engineering in the U.S. He made me realize how naive and old-fashioned I was in thinking that a three second delay could be crucial—in fact, he told me that a hundredth of a second could count just as in the speed records in the Olympic Games. Now it seemed that the financial warriors had less and less time to expertise the Itô Theory. Perhaps all of them have already internalized the Theory in their brains, and furthermore, can simultaneously use their brains, hands, and fingers. Imagining young people gaining or losing billions of dollars in a moment, my old-fashioned mind recalled Ryunosuke Akutagawa’s *Toshishun*, one of my favorite works as a child.

There was a time when, as a young boy, I was also a kind of math wizard in my hometown, someone who could solve mathematical problems faster than anyone around. So, had I not become a mathematician, perhaps I could have become a soldier of the economic war. Armed with the theories I described in my two earliest papers of

1942, I could perhaps have forecast the trends of the post-war black market and lived in luxury as a millionaire, or I could have lost my entire fortune and been standing by the Rakuyo gate, as poor and hungry as Toshishun, the hero of Akutagawa's novel.

As I told you earlier in this speech, I am against any war under any name. I would now like to include the "economic war." As I did not know exactly what "economy" was, I looked it up in a dictionary. "Economy" is defined as [1] to govern a country and save its people; [2] actions and processes of producing, distributing, and consuming physical properties that provide the basis for community life, and social relations that are formed through such actions processes. If "economy" represents such a noble concept, any war under the name of economy must be stopped as promptly as possible, and economic "soldiers" should be returned to the mathematics classrooms in their hometowns. Even if these soldiers are volunteers, they should leave the battlefield and return home, like Toshishun, who finally returned home after a dangerous and speculative life.

Each person goes his or her way as time passes, constantly influenced by random incidents. In which direction a person goes is determined by his set of values. Since mathematicians have fun reading and writing mathematical papers, they are *Homo ludens* (people who play) as well as *Homo loquens* (people who have a language).

Even though "play" is now recognized as essential for human beings, mathematicians and artists would not be able to enjoy their poetic lives, were it not for *Homo faber*, who produces not only tools, but also provide for the everyday necessities. It is *Homo faber* that has supported mathematicians and poets throughout human history. I would like to take this opportunity to express my appreciation to *Homo faber*, who has supplied various daily necessities and services for human societies, and my deepest thanks to my wife, who has supported me for over 60 years as my closest *Homo faber*.

For 60 years I have walked along the path of probability theory—the path of "stochastic processes" and "sample paths." Together with H. P. McKean, whom I first met in Princeton in 1954, I published *Diffusion Processes and Their Sample Paths* in 1965 from Springer Verlag in Germany. McKean and I spent more than ten years to complete this book. During this decade, both of us shuttled between American and Japanese universities to give lectures, and did various other work, but throughout the decade, we were grappling with the same problem. While we were working on this problem, however, we never realized the time passing by. Only when the work was over did we realize that a decade had passed.

Since I have always been either impetuous or absent-minded in daily life, it seems almost miraculous that I could carry out my research with care and persistence during these six decades. Of course, it is mainly due to those who provided me with the wonderful research environment over the years, but there may also be some influence from my father. My father was a high school teacher, but he was also a long-distance swimming champion. In his youth he often took part in long-distance ocean swimming competitions, covering more than 20 kilometers in 10 to 12 hours, or more than 40 kilometers in 24 hours. My classmates and I also learned long-distance swimming in junior high school, and by the time we graduated even the least athletic of us could easily swim over 5 kilometers.

This must have been so-called the good old leisurely days of the *Homo ludens*, when society permitted us to play and swim slowly but surely, covering huge distances. Today's society commands us to work and swim with high efficiency and speed. Perhaps mathematicians nowadays are not permitted to ponder over a single mathematical problem for a long time, watching clouds floating high in the sky. The speed of Tokyo, which surprised Sanshiro 86 years ago, and astonished me 60 years ago, continues to accelerate. Maybe poets and *Homo ludens* can find their true place only in a forest 20,000 years from now. Saying to myself, "It may be so. Yes, probably," I realized that this was "probability." The term "probability" or "*Wahrscheinlichkeit*" in German is commonly used in our daily lives. When we forecast tomorrow's weather, or try to figure out what is in the mind of a stray sheep, we do not know the truth, but we live on saying "Probably."

Such probability aspects in our daily lives contain some sentimentalism. Probability theory, however, is not sentimental. The history of probability theory originated in the correspondence between Pascal and Fermat in the 17th century, and ever since then until the 19th century there had been various disputes about the real meaning of probability, some of which were more philosophical than mathematical. Nowadays, however, the major concern of mathematicians is with laws of logic, that is, mathematical theorems that govern probability, and their applications to understand random phenomena.

I wish I could go on talking about Pascal, for I was invited to give a commemorative lecture at the Mathematical Colloquium on the Tricentenary of the Death of Blaise Pascal in Clermont-Ferrand, France in 1962. I had hoped to share with you some of my thoughts on these matters, but I am afraid I am running out of time. Writing the draft of this speech, I retraced the path along which I have walked with probability theory over the past 60 years, while I often wandered into the forest of time and space. Although I can never walk along the same path again, I am happy that I was able to trace the path in my mind. I have become a "thinking reed."

Thank you very much for your kind attention.