TIDY PARSIMONY

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We happy honorands were encouraged, in these commemorative lectures, to talk about ourselves. I remember myself as a small child sprawled on the floor and poring over my mother's old geography book. I aimlessly pondered North and South America, Europe, Africa. I neglected Asia, for the name was unfamiliar. Then one day I did happen to take a proper look at Asia, and the scales fell from my eyes. There were all those romantic names—Arabia, Jerusalem, Bagdad, Persia, India, China, Japan. Somehow I hadn't noticed their absence from the maps I had studied. They evidently had occupied the fairly-tale half of my brain. Now suddenly my world was one, and a rich one.

It was a purposeless pondering of boundaries, place names, and relative positions. It foreshadowed a taste for decisive distinctions and structure, as well as an almost but not quite insatiable wanderlust. In those early days I was given also to compiling lists, geographical and otherwise, to no better purpose than indulgence of a taste for tidy orderliness. It was a taste that was to favor mathematics and analytical philosophy over less disciplined disciplines.

It was a taste that took to algebra and geometry in school, and to the diagramming of English sentence, and to Latin. My responsiveness to languages had been whetted by stamp collecting, a hobby traceable to my interest in geography. German was unavailable until college because of the World War I, but I studied French.

Religion was not oppressive in my home, but it was there, and by the age of ten my doubts had prevailed over it. This surely is how many modern philosophers started up or down the philosophical path. Also I had, at about that age, a more specifically philosophical thought. Unfriendly remarks about Jews were not uncommon in my neighborhood, and two of my friends were Jews, which I regretted. Then it dawned on me that we should judge a class by its members.

My philosophical bent remained inarticulate, however, until college. I was just vaguely curious. I became actively interested rather in word origins and the history of language. I borrowed a book from the library on the subject and devoured it eagerly.

I have been speculating and checking on etymologies ever since.

At Oberlin College, consequently, I had to choose among three competing fields for my major subject: philosophy, philology, and mathematics. A friend told me that Bertrand Russell had something called mathematical philosophy, and that settled it. I majored in mathematics and arranged for honors reading in mathematical philosophy. Philology was outnumbered, two to one.

Mathematical philosophy turned out to be mathematical logic. It was not taught at Oberlin nor much elsewhere in America, but my professor got to me a reading list.

Practical mathematicians scoffed at mathematical logic as pedantic formalism. Mathematical logicians scoffed back fifteen years later, when their discipline had spawned general computer theory and become indispensable in programming. Meanwhile, in 1931, mathematical logic had enabled Kurt Gödel in Vienna to prove a theorem that revolutionized the philosophy of mathematics. By applying mathematical logic to itself, he proved that no explicit set of rules of proof can cover all mathematical truths, or cover even so limited part of mathematics as the theory of whole numbers. A proof procedure can always be strengthened but never enough, without getting some falsehoods. Before Gödel's discovery, we all thought each truth of mathematics could be proved, and proved by methods already at hand, though the proof might elude us. This, we thought, was what was distinctive about mathematics: truth is demonstrability. But not so.

It was two years before Gödel's theorem that I was at Oberlin reading Whitehead and Russell's great *Principia Mathematica*, where they show in three volumes that all of classical mathematics can be translated into a few symbols of mathematical logic. The objects that made up the universe of *Principia* were predominantly class. A class, for mathematics, is just any lot, finite or infinite, of objects of any sort, however unlike or remote from one another. In subsequent improvements on Whitehead and Russell's work, *all* the objects dealt with in pure classical mathematics end up as classes.

The numbers 0, 1, 2, etc. are an example. Each can be construed, however arbitrarily, as the class of all earlier ones. This makes 0 the empty class, 1 the class whose sole member is 0, 2 the class with the two members 0 and 1, and so on up. 0 has no members, 1 as one, 2 has two, and so on.

The three volumes of *Principia* were mostly in logical symbols. I reveled in the clarity, rigor, and elegance of the formulas and proofs and above all in the spectacular economy of the ideas that proved to suffice for the whole bewildering realm of classical mathematics. It was an achievement in tidy parsimony. A subsequent refinement by Gödel, and independently by Alfred Tarski in Poland, further enhanced the economy. They reduced the basic vocabulary to just the following. There are the adverb 'not' for negating a sentence and the conjunction 'and' for joining sentences. There is a generality prefix, with auxiliary variables, for saying that everything is thus and so. And finally, fourth, there is a verb 'is a member of' relating members to classes. I reduced these four basic devices to two equally simple ones. One is class inclusion, as in 'Dogs are animals.' The other is an abstruction prefix with auxiliary variables: 'the class of all objects such that.'

Whitehead and Russell took on the task in *Principia* not only of defining the various notions of classical mathematics, but also of framing axioms from which, along with the definitions, classical mathematics could be derived. At this point classes presented a deep problem: the paradoxes, the simplest of which is known as Russell's Paradox. It proceeds from the principle, which had long gone without saying, that every membership condition you can formulate determines a class, the class of all objects fulfilling the condition. Very well, says Russell, try this condition: '*x* is not a member of *x*.' This does not determine a class. There can be no such thing as the class of all non-self-members. It would belong to itself if and only it was a non-self-member. So we must rescind that obvious old rule. There are membership conditions that do not determine classes. This is one, and there are others.

But Russell did not rescind the old rule. He rejected the very words '*x* is not a member of *x*' from the language, along with other paradoxical membership conditions, by complicating the grammar. Such was his *theory of types*, which governed *Principia Mathematica*. Individuals comprised his lowest type, classes of individuals his second type, classes of such classes his third, and so on. Formulas were meaningless that affirmed membership otherwise than between objects of consecutive types.

A drawback of this expedient was that it saddled us with an infinite reduplication of arithmetic and the rest of mathematics, and of the logical class algebra itself, up the hierarchy of types. Each succeeding type had its universe class, its empty class, its numbers, all its mathematical ontology. With my predilection for tidy parsimony I deplored all this and sought less extravagant measures. I found that we could enjoy the protection conferred by Russell's high-handed restraints on grammar, and by his infinite reduplication of the mathematical world, while paying neither of these prices. Instead I gave up what Russell was preserving, namely the law that every membership condition determines a class. Then I noted what membership conditions had been rendered meaningless by Russell's restrictions on grammar, and just declared those sentences ineligible as membership conditions.

Along with its gains in simplicity, my system turned out to be stronger than Russell's in its production of classes. This raised suspicions of some lingering paradox in my system. I have since been busy with other things, but a number of bright mathematicians in Belgium, Switzerland, England, and America have sought paradox in it without success, while turning up various surprises along the way.

Ernst Zermelo in Germany had long since had his own way around the paradoxes, devised independently of Russell's and in the same year, 1908. Like me at my later date, he took the straightforward line of dropping the law that every membership condition determines a class. The laws that he then provided for existence of classes showed no kinship, as mine did, to Russell's theory of types. Zermelo's system, subsequently improved, is today's standard. The search down the years for a contradiction in my system has been coupled with counter-effort to establish its consistency by constructing a model of it within Zermelo's presumably consistent system. But this again has not succeeded.

A word now about the philosophical significance of the reduction of mathematics to logic, or to what has been called logic. It is a startling claim, for mathematics is proverbially mind-boggling whereas logic is proverbially obvious and trivial. The source of the confusion is the existence of classes, as is brought out by Russell's Paradox and the others. The paradoxes reveal class theory as by no means trivial, and rather as a desperate challenge; and mathematics depends on the existence of classes at almost every turn, with or without *Principia Mathematica*. The gulf between little old traditional logic and the theory of classes, known as set theory, is borne out also by Gödel's theorem, for that theorem applies to set theory along with number theory and higher branches.

The place to draw the boundary between logic and the rest of mathematics is at classes. What lies below that boundary is indeed as easy and trivial as the name suggests. What classical mathematics is reducible to is set theory, a formidable branch of mathematics in its own right. The reduction of mathematics to set theory is illuminating and exciting for the tidy parsimony that it yields, but there is no trivialization.

There are and have long been philosophers, called nominalists, who balk at the very existence of classes. There are sticks, stones, and all the other concrete objects, but nominalists draw the line at abstract objects, and classes are indeed abstract objects. Our abstract words contribute to the sentences in which they occur, the nominalists say, but are not names of abstract objects.

Another philosophical view of the matter is that once we get beyond words for concrete objects there is no real difference between viewing the word as naming and as not naming. I hold that both views come of looking in the wrong place. Where existence makes a difference is ordinarily not where we refer to a specific purported object, but where we are speaking of an unspecified object of a specified sort—some rabbit or other, some prime number—or every rabbit, every prime number. It is these repeated references to an identical but unspecified instance that introduce texture into our discourse and structure into our scientific theory. I go into detail in my workshop lecture.

Mathematics learns heavily on existence when existence is thus identified, and the existence leaned on is existence of numbers and other abstract objects, ultimately classes. Natural science in turn leans heavily on mathematics. Some philosophers profess nominalism by not heeding the commitments of their own day-to-day or scientific discourse: not considering what constitutes reference to abstract objects.

My recognition of abstract objects was a bit melancholy at first, but I have been fully reconciled to them on gaining a clearer view of the nature of the assuming of objects and the service they perform in the structure of scientific theory. However, my abstract objects are classes and only classes. They work wonders, providing, as I said, for numbers and everything else in mathematics. I do not concede existence to properties or to meanings, for these are in trouble over identity and difference. Two properties, it seems, can be properties of all and only the same things and yet be called different properties. Nor is there a clear account of what it takes in general for two expressions to count as having the same meaning. Tidy parsimony makes short shrift of all that.

There is an obvious confusion, carelessness basically, that has plagued thinkers even of the stature of Whitehead and Russell. It is confusion of the written word or sign with the object referred to. It happens only when the object is abstract. In expository parts of *Principia Mathematica* it muddies the thought of the authors and engenders needless complexities and obscurities. It is an evil—the confusion of use and mention—against which I have crusaded down the decades, with some success. I suspect that traces of it linger in the acquiescence of philosophers and layman in the notions of properties and meanings despite their infirmities in connection with identity. The philosopher who is out to clarify reality is ill advised to use notions as obscure as those he is trying to clarify. With classes, on the other hand, despite their abstractness, all is in order. They are as clearly identified as their members, for they are identical if they have the same members.

My own work in and about mathematical logic occupied most of my next twenty years after college and a few more recent ones. From mathematics at Oberlin I had proceeded to graduate work in philosophy at Harvard because of my admiration of Whitehead, who had been brought there as professor of philosophy after his retirement from mathematics in London. I found that the Harvard philosophers back then were happier than I with properties, meanings, propositions, necessity.

It was rather in Prague, on a postdoctoral fellowship two years later, that I first worked with an eminent philosopher who saw those matters as I did. He was Rudolf Carnap. I was similarly gratified on proceeding to Poland. I think it significant that both Carnap and the Poles were deep in mathematical logic. Sharpness of criteria and economy of assumptions—tidy parsimony—had guided them, as me. This is perhaps a basic contribution of mathematical logic to the philosophy of science, along with its direct and conspicuous contribution to the philosophy of mathematics. Whitehead and Russell, ironically, were perhaps too early to gain the full benefit of their own contribution.

My first five books, along with three later ones, were devoted to logic and set theory. I kept striving for shortcuts, for streamlining, for clearer formulations, with a view to making modern logic a routine acquisition of the general student. One minor venture to that purpose did prove useful to computer theory and has brought my name into computer manuals, though oddly enough I have never been lured to computers myself, even to the word processor.

Around age 45 I began to feel that I had done what I wanted to do in logic and set theory, though three of those eight logic books and three revised editions were still to come. I had been teaching a course in philosophy of science, inspired largely by Carnap, for fourteen years along with my teaching of logic and set theory. So my mind for the past forty years has been primarily on the philosophy of science.

I am concerned with our knowledge of the external world. Our intake from the world, in the way of information about what is going on around us, is just the triggering of our sensory receptors by the impact of light rays and molecules, plus some negligible kinaesthetic data. It is not much to go on. But we come out in the fullness of time with a torrential account of the world around us, out to the farthest nebula and down to the humblest quark.

Much of the interventing process was already prepared for by elaborate instincts, which are themselves accountable to natural selection down the generations. Instinctive standards of similarity implement the learning process. There is the development of language to account for, and the framing of hypotheses, and the testing of them by experiment. This is the domain of my workshop lecture.

The canons of neat precision and economy of assumptions―tidy parsimony―are as much to the point here in the philosophy of science as in the philosophy of logic and mathematics, and indeed they apply equally within natural science itself. What is so striking about the foundations of mathematics is just that it is there that those canons find the least impediment.